# Hash Join, Sort-Merge Join <br> Immanuel Trummer itrummer@cornell.edu www.itrummer.org 

## Database Management Systems (DBMS)


[RG, Sec. 12]

Connections, Security, Utilities, ...
Query Processor
Query Parser
Query Optimizer
Storage Manager


## Alternatives for Equality Joins?

## Hash Join

- Want tuples with same value in join column
- Same value in join column implies same hash value
- Join Phase 1
- Partition data by hash values in join columns
- Make partitions small enough to fit into memory
- Join Phase 2
- Join each partition pair (same hash value) separately


## More Notations

- Hash(Tuple): Calculates hash function for tuple
- Full(P): Whether page P has no more space left
- WriteAndClear(P): Write P to disk and erase


# Hash Join: Phase 1 

$\bowtie_{E . S i d=S . S i d}$

For ep in Pages(E):
LoadPage(ep)
For et in Tuples(ep):
Add et to EB[Hash(et)]
If (Full(EB[Hash(et)])):
WriteAndClear(EB[Hash(et)]))

# Hash Join: Phase 1 

$\bowtie_{E . S i d=S . S i d}$

For ep in Pages(E):
LoadPage(ep) « For each page in E
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# Hash Join: Phase 1 

$\bowtie_{E . S i d=S . S i d}$

For ep in Pages(E):
LoadPage(ep) « For each page in E
For et in Tuples(ep):
Add et to EB[Hash(et)]
If (Full(EB[Hash(et)])):
WriteAndClear(EB[Hash(et)])) $\longleftarrow$ For each page in E

$$
\text { Cost = pages in E* IO cost * } 2
$$

# Hash Join: Phase 1 

$\bowtie_{E . S i d=S . S i d}$

For sp in Pages(S):
LoadPage(sp) « For each page in S
For st in Tuples(sp):
Add st to SB[Hash(st)]
If (Full(SB[Hash(st)])):
WriteAndClear(SB[Hash(st)])) $\longleftarrow$ For each page in S

$$
\text { Cost }=\text { pages in } S^{*} I O \text { cost * } 2
$$

# Hash Join: Phase 2 

$\bowtie_{E . S i d=S . S i d}$
For h in Hash Values:
LoadPages(EB[h])
For sp in Pages(SB[h]):
Load(sp)
For ep in Pages(EB[h]), st in sp, et in ep:
If (et.Sid=st.Sid):
Output(et $\bowtie s t)$

# Hash Join: Phase 2 

$\bigotimes_{E . S i d=S . S i d}$
For h in Hash Values:
LoadPages $(E B[h]) \longleftarrow$ For each page in $E$
For sp in Pages(SB[h]):
Load(sp) $\longleftarrow$ For each page in S
For ep in Pages(EB[h]), st in sp, et in ep:
If (et.Sid=st.Sid):
Output(et $\bowtie$ st)

$$
\text { Cost }=(\text { pages in E in S) * IO cost }
$$

## How Much Memory?

- Phase 1
- Space to store current page read for partitioning
- Store one buffer page for each hash bucket
- Phase 2
- Store all pages from one hash bucket
- Store current page from other table bucket
- One output buffer page


## How Many Buckets?

- Constraint in Phase 1
- $1+$ Nr. Buckets $\leq$ Memory
- Constraint in Phase 2
- $2+$ Nr. Pages in Smaller Table/Nr. Buckets $\leq$ Memory
- Rule of thumb
- Want memory > Sqrt(Nr. Pages in Smaller Table)


## Example

## Property

## Value

## Enrollment Pages

Student Pages

Available Buffer

Hash Join Cost

1,000

11
Sqrt(100)<11
Cost: $3^{*}(100+1,000)$

## Details on Calculations

- Have enough buffer space to execute join as discussed
- Rule of thumb: $\operatorname{Sqrt}(100)=10<11$
- Phase 1 reads and writes each input table page once
- Cost is 2 * $(100+1,000)$
- Phase 2 reads and writes each input table page once
- However, we do not count the output cost, as usual
- Therefore, we only count cost 1 * $(100+1,000)$


## What If We Lack Memory?

- Number of buffer pages limits number of output buckets
- Not enough buckets means too much data per bucket
- Prevents us from loading one bucket entirely in Phase 2
- Hence, perform multiple passes over data in phase 1
- In each pass, buckets are partitioned into sub-buckets
- Iterate until data per bucket fits into main memory


## Sort-Merge Join: Idea

- Also specific to equality join conditions
- Phase 1 (Sort)
- Sort joined tables on the join column
- Phase 2 (Merge)
- Efficiently merge sorted tables together


## Join Phase 1: Overview

- Lots of sorting algorithms proposed in the literature
- However, typically assume that we access single entries
- But random data access can be very inefficient
- Hence, want to access pages of entries instead
- Need specialized ("external") sort algorithms


## Algorithm Sketch

- Step 1: load chunk of data and sort, write back to disk
- Step 2 .. n: merge sorted runs to produce larger runs
- Each merging step reduces number of runs (but longer)
- Finally, have only one sorted run left - we're done!


## Details on Step 1

- Assume we have B buffer pages available
- Load chunks of B pages into the buffer
- For each chunk, sort by standard sort algorithm
- Can use standard algorithm as all data in memory
- Then, write sorted data to hard disk
- A sorted sequence of data is called a "run"


## Step 1 Example



Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 12,29 | 9,10 | 15,3 | 26,4 | 14,17 | 19,54 | 8,90 | 6,12 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

| 1,8 | 12,29 | 9,10 |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 12,29 | 9,10 | 15,3 | 26,4 | 14,17 | 19,54 | 8,90 | 6,12 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

| 1,8 | 9,10 | 12,29 |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

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## Step 1 Example

| 1,8 | 9,10 | 12,29 |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

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| 1,8 | 9,10 | 12,29 | 15,3 | 26,4 | 14,17 | 19,54 | 8,90 | 6,12 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

| 1,8 | 9,10 | 12,29 |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 15,3 | 26,4 | 14,17 | 19,54 | 8,90 | 6,12 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

$$
\begin{array}{l|l|l}
15,3 & 26,4 & 14,17
\end{array}
$$

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 15,3 | 26,4 | 14,17 | 19,54 | 8,90 | 6,12 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

| 3,4 | $14,15 \quad 17,26$ |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 15,3 | 26,4 | 14,17 | 19,54 | 8,90 | 6,12 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

| 3,4 | $14,15 \quad 17,26$ |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 19,54 | 8,90 | 6,12 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

$$
19,54 \quad 8,90 \quad 6,12
$$

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 19,54 | 8,90 | 6,12 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

$6,8 \quad 12,19 \quad 54,90$

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 19,54 | 8,90 | 6,12 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

| 6,8 | 12,19 | 54,90 |
| :---: | :---: | :---: |

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

$6,8 \quad 12,19 \quad 54,90$

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

| 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

| 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 5,73 | 2,42 | 3,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 1 Example

| 2,3 | 3,5 | 9,73 |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

## Hard Disk (12 Pages)

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Details on Steps 2 .. n

- (Still have B buffer pages available)
- Enables us to merge B-1 sorted runs into one in one step
- Load first page of each sorted run into $B-1$ pages
- Copy minimum entry in input buffers to output buffer
- If output buffer full, write to disk and clear
- Erase minimum entry from input buffer
- If input buffer becomes empty, load next page


## Step 2 Example

Input 1 Input 2 Output


Buffer Pool (3 Pages)

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

# Step 2 Example <br> Input 1 Input 2 Output 



Buffer Pool (3 Pages)

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | 42,73

## Step 2 Example

Input 1 Input 2 Output

| 1,8 | 3,4 |
| :--- | :--- |

Buffer Pool (3 Pages)

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 2 Example

Input 1 Input 2 Output
$8 \quad 3,4$
1

Buffer Pool (3 Pages)

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 2 Example

Input 1 Input 2 Output

| 8 | 4 | 1,3 |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 2 Example

Input 1 Input 2 Output

| 8 | 4 |
| :--- | :--- | :--- |

Buffer Pool (3 Pages)

1,3

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 42,73 |  |  |  |  |  |  |  |  |  |  |

## Step 2 Example

Input 1 Input 2 Output

| 8 | 4 |
| :--- | :--- |

Buffer Pool (3 Pages)

1,3

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 42,73 |  |  |  |  |  |  |  |  |  |  |

## Step 2 Example

Input 1 Input 2 Output
$8 \quad 14,15 \quad 4$

Buffer Pool (3 Pages)

1,3

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 2 Example

Input 1 Input 2 Output

$$
14,15 \quad 4,8
$$

Buffer Pool (3 Pages)

1,3

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 2 Example

Input 1 Input 2 Output

14, 15

## Buffer Pool (3 Pages)

$1,3 \quad 4,8$

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 2 Example

Input 1 Input 2 Output

$$
9,10 \quad 14,15
$$

Buffer Pool (3 Pages)
$1,3 \quad 4,8$

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 2 Example

Input 1 Input 2 Output
$10 \quad 14,15 \quad 9$

Buffer Pool (3 Pages)
$1,3 \quad 4,8$

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 2 Example

Input 1 Input 2 Output

$$
14,15 \quad 9,10
$$

Buffer Pool (3 Pages)
$1,3 \quad 4,8$

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Step 2 Example

Input 1 Input 2 Output

12, 29 14, 15

Buffer Pool (3 Pages)


## Step 2 Example

Input 1 Input 2 Output
29 14, $15 \quad 12$

Buffer Pool (3 Pages)
$1,3 \quad 4,8 \quad 9,10$

## Hard Disk

| 1,8 | 9,10 | 12,29 | 3,4 | 14,15 | 17,26 | 6,8 | 12,19 | 54,90 | 2,3 | 5,9 | 42,73 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

## Example Summary

- Have 12 pages to sort with 3 buffer pages
- First step: produce 4 sorted runs of length 3
- Can merge 2 runs in each merge step
- Second step: produce 2 sorted runs of length 6
- Third step: produce 1 sorted run of length 12


## Cost Analysis (Phase 1)

- Multiple sorting passes, we read and write data once in each
- Cost per pass is 2 * N ( N is number of pages)
- How many steps must we make with B buffer pages?
- First step produces runs of length B
- Second step produces runs of length (B-1) * $B$
- Third step produces runs of length (B-1) * $(B-1)$ * $B$...
- Stop once (B-1) $)^{\text {steps-1* }} \mathbf{B} \geq \mathbf{N}$, after $1+$ Ceil( $\left(\log _{B-1}(N / B)\right)$ steps


## Join Phase 2: Overview

- (Have sorted both input tables by their join column)
- Load first page of both sorted tables into memory
- Find matching tuples and add to join result output
- Load next page for table with smallest last entry
- Keep doing until no pages left for one table


## Join Phase 2 Example

Input 1 Input 2 Output


Buffer Pool (3 Pages)

## Hard Disk

1,3
4, 6
8, 9
12, 14
$15,1726,29$
31, 32
45, 50


Table 1

## Join Phase 2 Example

Input 1 Input 2 Output

$$
1,3 \quad 2,9
$$

Buffer Pool (3 Pages)

## Hard Disk

1,3
4, 6
8, 9
12, 14
$15,1726,29$
31, 32
45,50


Table 1

## Join Phase 2 Example

Input 1 Input 2 Output
$3 \quad 2,9$

Buffer Pool (3 Pages)

## Hard Disk

1, 3
4, 6
8, 9
12, 14
$15,1726,29$
31, 32
45,50


Table 1

## Join Phase 2 Example

Input 1 Input 2 Output
$3 \quad 9$

Buffer Pool (3 Pages)

## Hard Disk

1,3
4, 6
8, 9
12, 14
$15,1726,29$
31, 32
45, 50


Table 1

## Join Phase 2 Example

Input 1 Input 2 Output


Buffer Pool (3 Pages)

## Hard Disk

1,3
4, 6
8, 9
12, 14
$15,1726,29$
31, 32
45, 50


Table 1

## Join Phase 2 Example

Input 1 Input 2 Output
$4,6 \quad 9$

Buffer Pool (3 Pages)

## Hard Disk



## Join Phase 2 Example

Input 1 Input 2 Output
$6 \quad 9$

Buffer Pool (3 Pages)

## Hard Disk



## Join Phase 2 Example

Input 1 Input 2 Output


Buffer Pool (3 Pages)

## Hard Disk



## Join Phase 2 Example

Input 1 Input 2 Output
$8,9 \quad 9$

Buffer Pool (3 Pages)

## Hard Disk



## Join Phase 2 Example

Input 1 Input 2 Output
$9 \quad 9$

Buffer Pool (3 Pages)

## Hard Disk



## Join Phase 2 Example

Input 1 Input 2 Output


Buffer Pool (3 Pages)

## Hard Disk



## Join Phase 2 Example

Input 1 Input 2 Output<br>$$
\begin{array}{l|l|l}
12,14 & 16,25 & 9
\end{array}
$$

Buffer Pool (3 Pages)

## Hard Disk



## Join Phase 2 Example

Input 1 Input 2 Output
14 16, $25 \quad 9$

Buffer Pool (3 Pages)

## Hard Disk



## Join Phase 2 Example

| Input 1 Input 2 | Output |
| ---: | ---: | ---: |
| 16,25 | 9 |

Buffer Pool (3 Pages)

## Hard Disk



## Join Phase 2 Example

Input 1 Input 2 Output<br>$$
15,17 \quad 16,25 \quad 9
$$

Buffer Pool (3 Pages)


## Join Phase 2 Example

Input 1 Input 2 Output
17 16, $25 \quad 9$

Buffer Pool (3 Pages)


## Join Phase 2 Example

Input 1 Input 2 Output
$17 \quad 25 \quad 9$

Buffer Pool (3 Pages)


## Join Phase 2 Example

Input 1 Input 2 Output
25
9

Buffer Pool (3 Pages)


## Join Phase 2 Example

| Input 1 Input 2 | Output |  |
| ---: | ---: | ---: |
| 26,29 | 25 | 9 |

Buffer Pool (3 Pages)


## Join Phase 2 Example

| Input 1 Input 2 Output |  |
| ---: | :---: |
| 26,29 | 9 |

Buffer Pool (3 Pages)


## Join Phase 2 Example

Input 1 Input 2 Output<br>$$
26,2930,90 \quad 9
$$

Buffer Pool (3 Pages)


## Join Phase 2 Example

Input 1 Input 2 Output
$2930,90 \quad 9$

Buffer Pool (3 Pages)


## Handling Many Duplicates

- May have duplicates over multiple pages
- Duplicate entry: same value in join column
- Must revert to first page with duplicate whenever we load new page from other table
- This makes the join more expensive


## Cost Analysis (Phase 2)

- For now: assume all duplicate entries on same page
- Means that each input page is only read once
- Cost is proportional to number of input pages
- I.e., Pages from both input tables


## Total Join Cost

- Two input tables with M and N pages, B buffer pages
- First phase has cost
- $2 * \mathbf{M}^{*}\left(1+\right.$ Ceill $^{\left.\left(\log _{\mathrm{B}-1}(\mathrm{M} / \mathrm{B})\right)\right)}$ for sorting table 1
- 2* $\mathbf{N}^{*}\left(1+\right.$ Ceil $^{\left.\left(\log _{\mathrm{B}-1}(\mathrm{~N} / \mathrm{B})\right)\right)}$ for sorting table 2
- Second phase has cost
- M+N (we don't count cost for writing output!)


## How Much Memory?

- First phase: try to exploit all buffer pages
- More buffer means less merging passes!
- Second phase: only exploit three buffer pages
- One for first input, one for second input, one output


## How Much Memory?

- First phase: try to exploit all buffer pages
- More buffer means less merging passes!
- Second phase: only explo three buffer pages
- One for first input, one for second in sub-Optimal! seems Sub


## Refined Sort-Merge Join

- Idea: can merge more than two sorted tables in phase 2
- Hence, do not need to sort tables completely in phase 1
- Means we can save steps (i.e., passes over the data)
- First phase: only sort data chunks that fit into memory
- Second phase: join all sorted chunks together (one step)


## Refined Join Details

- Assume B buffer pages, tables with N and M pages
- First phase: load chunks of B pages, sort, write back
- We now have (N+M)/B sorted chunks on disk
- Second phase: merge B-1 sorted chunks together
- Can sort entries in-memory to find matches
- Cost is $2^{*}(\mathrm{M}+\mathrm{N})($ Phase 1$)+1^{*}(\mathrm{M}+\mathrm{N})$ (Phase 2$)$


## How Much Memory?

- Again, B buffer pages, input sizes are M and N
- Have (N+M)/B sorted runs after first phase
- Need $\mathrm{B}-1 \geq(\mathrm{N}+\mathrm{M}) / \mathrm{B}$ to merge them in one step
- Rule of thumb if $\mathrm{N}>\mathrm{M}$ : need $\mathrm{B} \geq 2^{*} \operatorname{Sqrt}(\mathrm{~N})$


## R-SMJ vs. Hash Join

|  | Hash Join | Refined <br> Sort-Merge Join |
| :---: | :---: | :---: |
| Time | 3 * Input Size | 3 * Input Size |
| Memory | > Sqrt(Smaller <br> Table Size) | $>2$ * Sqrt(Larger <br> Table Size) |
| Parallelization | Advantage |  |
| Skew-Resistance |  | Advantage |

